NUMBERS IN REGIONS

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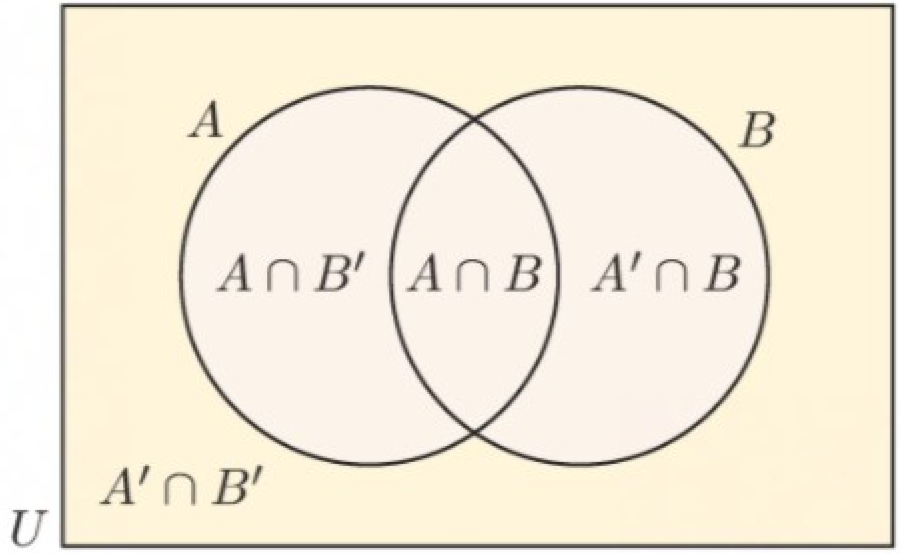
We have seen that there are four regions on a Venn diagram which contains two overlapping sets A and B.



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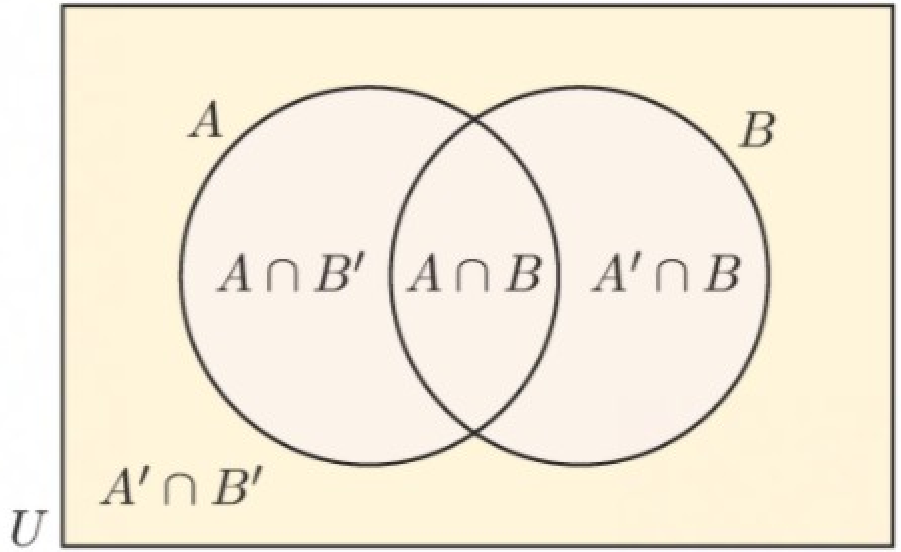
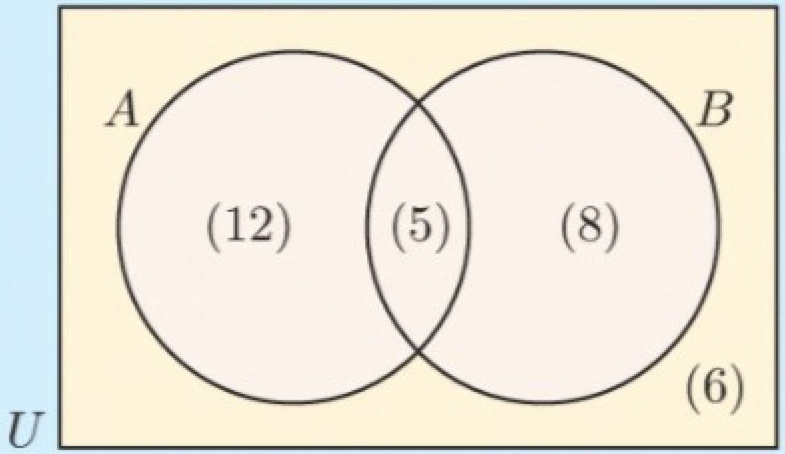


There are many situations where we are only interested in the **number of elements** of U that are in each region. We do not need to show all the elements on the diagram, so instead we write the number of elements in each region in brackets.

Example 10

Self Tutor



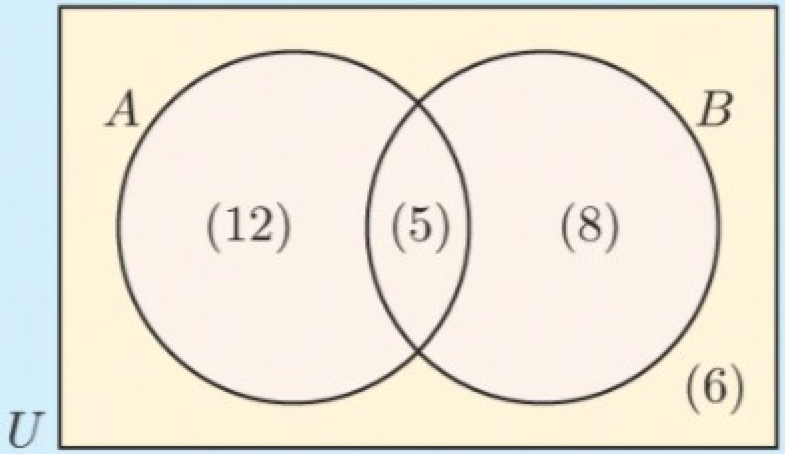
In the Venn diagram given, (5) means that there are 5 elements in the set $A \cap B$.

How many elements are there in:

a A **b** B' **c** $A \cup B$

d A, but not B **e** B, but not A **f** neither A nor B?

Example 10 **Self Tutor**



In the Venn diagram given, (5) means that there are 5 elements in the set $A \cap B$.

How many elements are there in:

a A **b** B' **c** $A \cup B$
d A, but not B **e** B, but not A **f** neither A nor B?

a $n(A) = 12 + 5 = 17$

b $n(B') = 12 + 6 = 18$

c $n(A \cup B) = 12 + 5 + 8 = 25$

d $n(A, \text{ but not } B) = 12$

e $n(B, \text{ but not } A) = 8$

f $n(\text{neither } A \text{ nor } B) = 6$

Example 11

Self Tutor

Given $n(U) = 25$, $n(P) = 10$, $n(Q) = 12$, and $n(P \cap Q) = 3$, find:

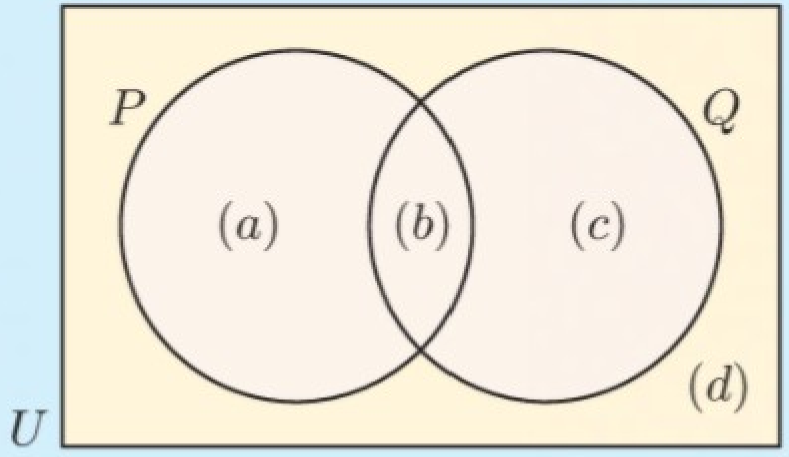
a $n(P \cup Q)$

b $n(P$, but not $Q)$

Example 11 **Self Tutor**

Given $n(U) = 25$, $n(P) = 10$, $n(Q) = 12$, and $n(P \cap Q) = 3$, find:

a $n(P \cup Q)$ **b** $n(P, \text{ but not } Q)$



We see that $b = 3$ {as $n(P \cap Q) = 3$}

$a + b = 10$ {as $n(P) = 10$}

$b + c = 12$ {as $n(Q) = 12$}

$a + b + c + d = 25$ {as $n(U) = 25$}

$\therefore\ b = 3,\ a = 7,$ and $c = 9$

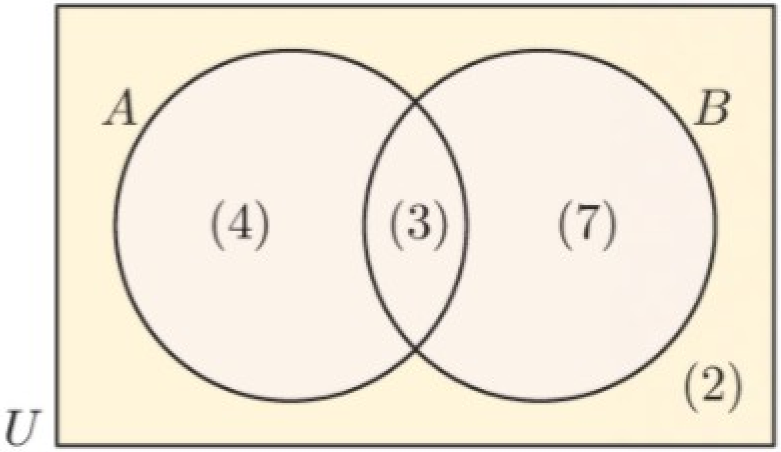
$\therefore\ 7 + 3 + 9 + d = 25$

$\therefore\ d = 6$

a $n(P \cup Q) = a + b + c = 19$

b $n(P, \text{ but not } Q) = a = 7$

1



How many elements are there in:

a B

b A'

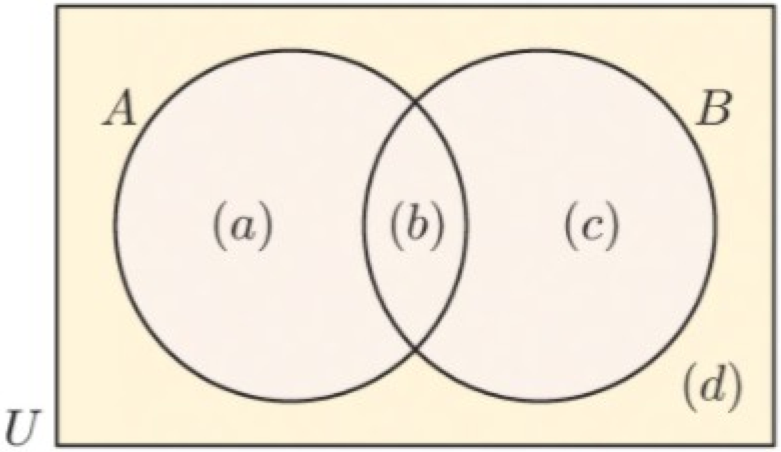
c $A \cup B$

d A, but not B

e B, but not A

f neither A nor B?

2 In the Venn diagram below, (a) means that there are a elements in that region.



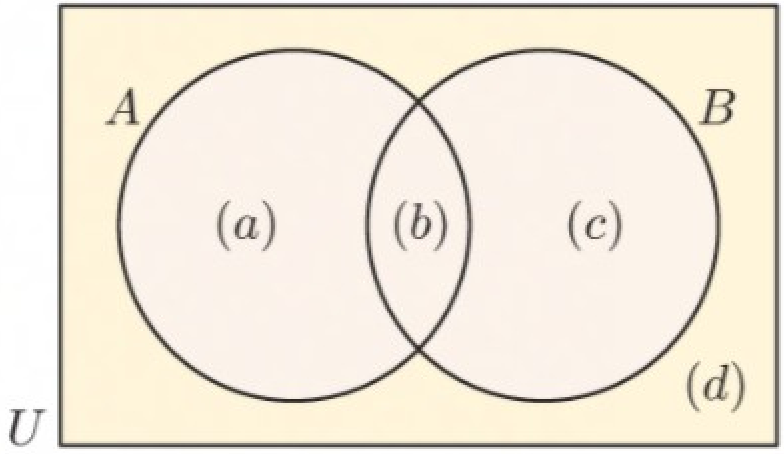
a Write an expression for:

- **i** $n(A)$
- **ii** $n(B)$
- **iii** $n(A \cap B)$
- **iv** $n(A \cup B)$

b Show that:

- **i** $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
- **ii** $n(A \cap B) = n(A) + n(B) - n(A \cup B)$
- **iii** if A and B are disjoint, then $n(A \cup B) = n(A) + n(B)$.

3



Use the Venn diagram to show that:

a $n(A \cap B') = n(A) - n(A \cap B)$

b $n(A \cup B') = n(U) - n(A' \cap B)$

4 Given $n(U) = 20$, $n(A) = 12$, $n(B) = 13$, and $n(A \cap B) = 8$, find:

a $n(A \cup B)$

b $n(B$, but not $A)$

5 Given $n(U) = 28$, $n(M) = 14$, $n(M \cap N) = 3$, and $n(M \cup N) = 18$, find:

a $n(N)$

b $n((M \cup N)')$

1 **a** 10 **b** 9 **c** 14 **d** 4 **e** 7 **f** 2

2 **a** **i** $n(A) = a + b$ **ii** $n(B) = b + c$

iii $n(A \cap B) = b$ **iv** $n(A \cup B) = a + b + c$

b **i**
$$\begin{aligned} n(A) + n(B) - n(A \cap B) &= a + b + b + c - b \\ &= a + b + c \\ &= n(A \cup B) \end{aligned}$$

ii
$$\begin{aligned} &n(A) + n(B) - n(A \cup B) \\ &= a + b + b + c - (a + b + c) \\ &= a + 2b + c - a - b - c \\ &= b \\ &= n(A \cap B) \end{aligned}$$

iii If A and B are disjoint then $n(A \cap B) = b = 0$.

$$\begin{aligned} \text{So,} \quad n(A \cup B) &= a + b + c \\ &= a + c \end{aligned}$$

$$\begin{aligned} \text{and} \quad n(A) + n(B) &= a + b + b + c \\ &= a + c \\ &= n(A \cup B) \end{aligned}$$

3 **a** $n(A) - n(A \cap B) = (a + b) - b$

$$= a$$

$$\therefore \quad n(A \cap B') = n(A) - n(A \cap B)$$

b $n(U) - n(A' \cap B) = (a + b + c + d) - c$

$$= a + b + d$$

$$\therefore \quad n(A \cup B') = n(U) - n(A' \cap B)$$

4 **a** 17 **b** 5 **5** **a** 7 **b** 10